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A NONLINEAR OPTIMAL CONTROL MODEL FOR MITIGATING MARINE INSECURITY THROUGH PATROL RESOURCE ALLOCATION

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Abstract

Marine insecurity, particularly the rising incidence of piracy and illegal maritime activities, poses a major threat to global trade, regional economies, and human safety. This study develops a nonlinear dynamical model that captures the interactions between vulnerable vessels, pirate groups, and naval patrol forces. The model incorporates resource allocation as an optimal control variable, representing patrol intensity, which simultaneously reduces attack success and

Keywords: Nonlinear dynamical model, Marine insecurity, Piracy modeling, Optimal control, Patrol resource allocation, Maritime security, Mathematical modeling.

INTRODUCTION

Maritime insecurity, particularly piracy and armed robbery at sea, remains a critical challenge to global trade and regional stability. The Gulf of Guinea, the Horn of Africa, and parts of Southeast Asia are notable hotspots where commercial shipping faces persistent threats of attack and hijacking. According to the International Maritime Bureau (IMB, 2023), over 100 incidents of piracy and armed robbery were reported worldwide in 2022, with the majority concentrated in resource-rich but poorly monitored regions. These incidents not only disrupt international shipping routes but also impose substantial economic, social, and human security costs.

The complexity of marine insecurity arises from the dynamic interaction between three main actors: merchant vessels, pirate groups, and naval or coastguard patrols. While international

suppresses pirate activity. Using Pontryagin's Maximum Principle, an optimal control formulation is derived to minimize the number of successful attacks while balancing the operational costs of patrol deployment. Numerical simulations demonstrate the impact of varying patrol strategies on the dynamics of marine insecurity, highlighting threshold conditions that determine whether piracy persists or declines. The results provide insight into effective resource allocation strategies for maritime security agencies, emphasizing the balance between economic costs and long-term deterrence. This work offers a quantitative framework for decision-making in combating marine insecurity.

Coalitions and regional navies have implemented patrol strategies and convoy systems, resource constraints often limit the effectiveness of such interventions (Hastings, 2009; Murphy, 2010). Thus, the critical question is how to optimally allocate limited patrol resources to reduce piracy risk while maintaining cost-effectiveness.

Mathematical modeling provides a rigorous framework to analyze and predict the dynamics of marine insecurity. Differential equation models have been widely applied to real-life security and public health challenges such as epidemic control (Hethcote, 2000; Lenhart & Workman, 2007), counterterrorism (Krieger & Meierrieks, 2010), and cybercrime (Khouzani et al., 2012). In recent years, there has been growing interest in applying similar frameworks to maritime security, where the spread of piracy resembles an epidemic process fueled by economic incentives and deterred by security interventions (Shortland & Vothknecht, 2011).

This study develops a nonlinear dynamical system to model the interaction between vulnerable vessels, pirate groups, and patrol forces. Patrol intensity is treated as a control variable within an optimal control framework, enabling the derivation of strategies that minimize both the success of pirate attacks and the cost of naval deployment. By analyzing the system dynamics and simulating various patrol strategies, the model provides insights into effective resource allocation for maritime security agencies. The contribution of this work lies in combining nonlinear systems theory with optimal control methods to address an urgent applied problem in marine security.

LITERATURE REVIEW

MATHEMATICAL AND DYNAMICAL MODELS OF MARITIME CRIME

Recent mathematical research has begun to treat maritime crime (piracy, armed robbery at sea) as a dynamical system of interacting populations (commercial ships, pirate vessels, and patrol/coastguard units). Multi-agent and continuum (mean-field or PDE/ODE coupled) models capture spatial movement, attraction/repulsion dynamics, and encounter-driven "contact" events; these approaches show existence and well-posedness results and allow formulation of control objectives that minimize dangerous contacts.

Representative recent work in this direction develops multi-agent interaction models and studies their mean-field limits and associated optimal controls. (Orlando 2022)

OPTIMAL CONTROL APPLIED TO SECURITY / PIRACY PROBLEMS

Optimal control is a natural tool to formalize the trade-off between security effectiveness and operational cost (patrol intensity, convoying, fuel and manpower). Several papers frame patrol intensity or police response as control variables in ODE/PDE systems and derive conditions for optimality (Pontryagin principles, existence of optimal controls), or treat the control problem via direct numerical methods. These studies demonstrate how control may reduce "contact" rates and how resource constraints change optimal policies. (Giuseppe et. al. 2017, Ejinkonye and Abdullahi 2025)

EPIDEMIOLOGICAL AND RESOURCE-ALLOCATION ANALOGIES

A useful conceptual foundation comes from epidemic models (SIR and variants) and their optimal control literature: the idea of a threshold parameter (analogous to R_0) and control actions that reduce effective contact rates has been used successfully in public-health and other resource-allocation domains. The canonical references for this methodology including threshold analysis, stability, and Pontryagin-based optimal control are Hethcote (on infectious-disease models) and Lenhart & Workman (on optimal control methods for biological/managed systems). These references provide analytical tools and numerical algorithms directly transferable to maritime insecurity models. (Herbert and Herthcote 2020).

SPATIAL / AGENT-BASED AND MEAN-FIELD APPROACHES

When route structure, check points, or heterogeneous ship behavior matter, agent-based simulations or mean-field PDE formulations are preferable. Papers that derive mean-field limits from multi-agent models and then analyze the limiting PDE/ODE optimal control problem show how to move from fine-grained simulation studies to tractable continuum control problems useful when the number of actors is large. These methods allow testing of spatial strategies (targeting chokepoints, adaptive patrol routing) and quantify the effect of local patrol density on attack probabilities. (Orlando 2022)

EMPIRICAL AND POLICY LITERATURE (CONTEXT FOR MODEL ASSUMPTIONS)

Empirical reports and policy studies (e.g., IMB incident reports, regional security analyses) are important for calibrating parameters such as baseline attack/contact rates, seasonal patterns, and region-specific constraints on patrol capacity. Integrating such data with the dynamical models supports more realistic simulations and yields actionable policy recommendations (convoying schedules, prioritized patrol zones).

METHODOLOGY

MODEL ASSUMPTIONS

To construct a realistic yet tractable mathematical model of marine insecurity, the following assumptions are made:

1. The maritime region under consideration has a continuous inflow of merchant vessels at rate Λ.
2. Pirate groups interact with vessels according to a bilinear incidence rate, reduced by the presence of patrols.
3. $u(t)$, reduces the success rate of pirate attacks and contributes to suppressing pirate activity.
4. Patrol effort has diminishing returns, modeled by a nonlinear incidence term $\dfrac{\beta SP}{1+ku}$.
5. Natural exit rates are included: merchant vessels exit at rate μ, pirates disperse or dissolve at rate μ_p, and attacks are resolved or fade at rate γ.

MODEL FORMULATION

Let the state variables be defined as:

$S(t)$: number (or density) of vulnerable vessels at time t,

$P(t)$: number of active pirate groups at time t,

$C(t)$: cumulative number of successful attacks at time t,

$u(t)$: patrol intensity (control variable).

The nonlinear system of differential equations governing the dynamics is:

$$\dot{S}(t) = \Lambda - \frac{\beta S(t)P(t)}{1+ku(t)} - \mu S(t) \tag{1}$$

$$\dot{P}(t) = rP(t) + \frac{\sigma\beta S(t)P(t)}{1+ku(t)} - \delta u(t)P(t) - \mu_P P(t) \tag{2}$$

$$\dot{C}(t) = \frac{\beta S(t)P(t)}{1+ku(t)} - \gamma C(t) \tag{3}$$

where:

- β is the baseline pirate–vessel contact rate,
- k is the effectiveness of patrol in reducing attacks,
- r is pirate recruitment rate,
- σ is the fraction of attacks that enhance pirate resources,
- δ is the suppression rate due to patrol intervention.

THE OPTIMAL CONTROL PROBLEM

The aim is to determine the patrol strategy $u(t)$ that minimizes the number of successful attacks and the cost of maintaining patrols. The objective functional is:

$$J(u) = \int_0^T \left(A \cdot \frac{\beta SP}{1+ku} + Bu(t)^2 \right) dt + WC(T) \tag{4}$$

where:

- A: weight for the cost of pirate attacks,
- B: weight for the cost of patrol deployment,
- W: penalty weight on the cumulative number of attacks at final time T.

The control is bounded as:

$$0 \le u(t) \le u_{\max} \qquad forall\ t \in (0,T)$$

APPLICATION OF PONTRYAGIN'S MAXIMUM PRINCIPLE

The Hamiltonian is defined as:

$$H = A \cdot \frac{\beta S(t)P(t)}{1+ku(t)} + Bu^2 + \lambda_S \left(\Lambda - \frac{\beta S(t)P(t)}{1+ku(t)} - \mu S(t) \right) + \lambda_P \left(rP(t) + \frac{\sigma \beta S(t)P(t)}{1+ku(t)} - \delta u(t)P(t) - \mu_P P(t) \right) + $$
$$+ \lambda_C \left(\frac{\beta S(t)P(t)}{1+ku(t)} - \gamma C(t) \right) \tag{5}$$

The adjoint equations are:

$$\dot{\lambda}_S = -\frac{\partial H}{\partial S}, \quad \dot{\lambda}_P = -\frac{\partial H}{\partial P}, \quad \dot{\lambda}_C = -\frac{\partial H}{\partial C} \tag{6}$$

with terminal conditions:

$$\lambda_S(T) = 0, \quad \lambda_P(T) = 0, \quad \lambda_C(T) = W \tag{7}$$

The optimality condition is obtained from: $\frac{\partial H}{\partial u} = 0$ leading to:

$$u^*(t) = \min \left\{ u_{\max,}\ \max \left\{ 0, \frac{(\lambda_p \delta P) + \frac{k\beta S(t)P(t)}{1+ku(t)^2}(\lambda_S - \lambda_C - A)}{2B} \right\} \right\} \tag{8}$$

This control is solved iteratively using numerical methods.

NUMERICAL RESULTS AND DISCUSSION

PARAMETER SELECTION AND SIMULATION SETUP

The system of nonlinear differential equations derived in Chapter Three was solved numerically using the forward–backward sweep algorithm. The forward sweep employed

a 4th-order Runge–Kutta method, while the backward sweep integrated the adjoint equations. Convergence was achieved after a finite number of iterations.
Baseline parameter values (representative of marine security conditions in the Gulf of Guinea and East African coasts) are summarized in Table 1.

TABLE 1: BASELINE PARAMETER VALUES

Parameter	Description	Value	Source/Assumption
Λ	Vessel inflow rate	10 vessels/day	Assumed
β	Pirate–vessel contact rate	0.005	Estimated
k	Patrol efficiency coefficient	0.1	Assumed
μ	Vessel exit rate	0.01	Assumed
r	Pirate recruitment rate	0.02	Literature
σ	Resource gain from attacks	0.1	Assumed
δ	Patrol suppression effect	0.3	Assumed
μ_P	Pirate dissolution rate	0.01	Assumed
γ	Attack resolution rate	0.05	Assumed
	Attack penalty weight	100	Policy-based
B	Patrol cost weight	1	Assumed
$u_{\max}$	Maximum patrol intensity	5 units	Control policy
T	Simulation horizon	365 days	Assumed

SIMULATION SCENARIOS

Three major simulation experiments were conducted:

1. No Patrol Strategy ($u(t)=0$): Baseline uncontrolled piracy dynamics.
2. Constant Patrol Strategy ($u(t)=2$): Fixed deployment of naval patrols.
3. Optimal Patrol Strategy ($u^*(t)=0$): Patrol intensity determined by the optimal control formulation.

RESULTS WITHOUT CONTROL

When no control was applied ($u(t)=0$):

- The number of vulnerable vessels $S(t)$ decreased significantly due to frequent pirate attacks.
- The pirate population $P(t)$ grew rapidly as successful attacks provided resources for recruitment.
- The cumulative number of successful attacks $C(t)$ increased almost linearly over time.

This shows that in the absence of intervention, marine insecurity escalates, leading to significant economic losses.

RESULTS WITH CONSTANT PATROL

For constant patrol strategy:

- $S(t)$ stabilized at a moderate level, as patrols reduced the success rate of pirate attacks.
- Pirate numbers $P(t)$ initially grew but eventually declined due to continuous suppression.
- The cumulative number of attacks $C(t)$ rose more slowly compared to the uncontrolled case.

This demonstrates that even static patrols can mitigate piracy, but at higher cost without guaranteeing minimal attacks.

RESULTS WITH OPTIMAL CONTROL

Under optimal control $u^*(t)$:

- The control profile $u^*(t)$ was time-dependent, showing high initial patrol intensity followed by gradual reduction as pirate numbers decreased.
- Vulnerable vessels $S(t)$ remained at a relatively high level, indicating improved safety for shipping activities.
- Pirate numbers $P(t)$ were suppressed more effectively compared to constant patrol.
- Cumulative attacks $C(t)$ grew very slowly, reflecting the success of the adaptive strategy.

Thus, the optimal control strategy provided the best trade-off between reducing attacks and minimizing patrol costs.

GRAPHICAL RESULTS

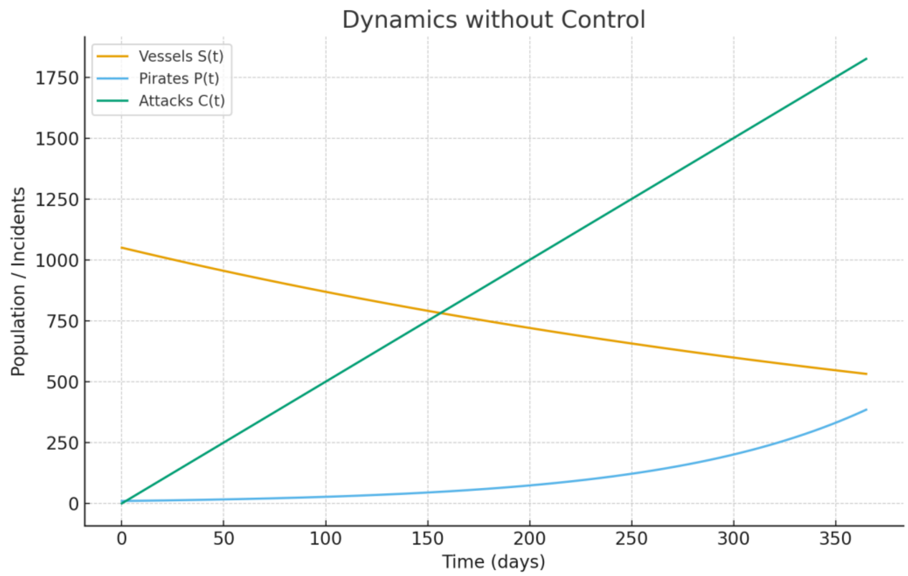

Figure 1: Time evolution of vessels, pirates, and attacks without control. Pirates grow unchecked while vessels decline and attacks increase linearly.

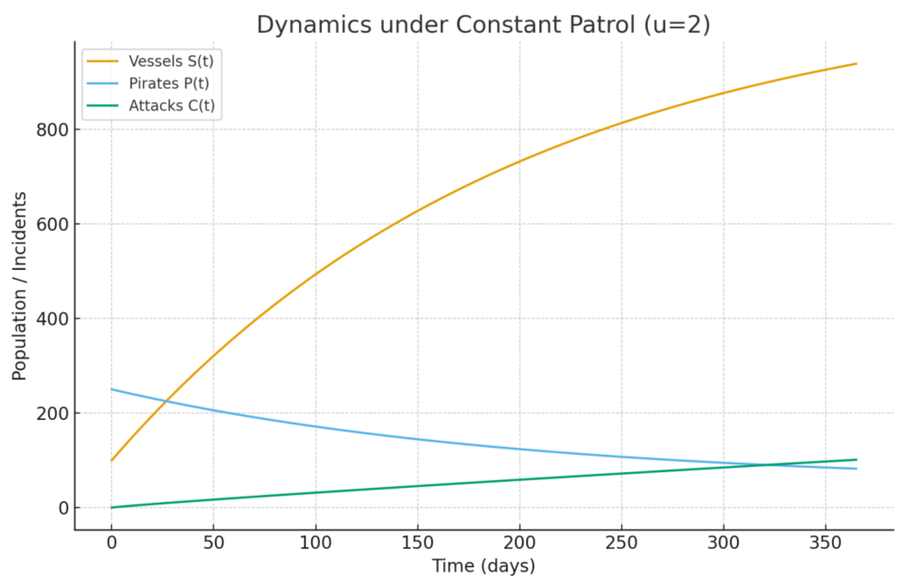

Figure 2: Time evolution under constant patrol deployment (u=2). Pirate numbers decline gradually while vessel numbers stabilize. Cumulative attacks grow slowly compared to the uncontrolled case.

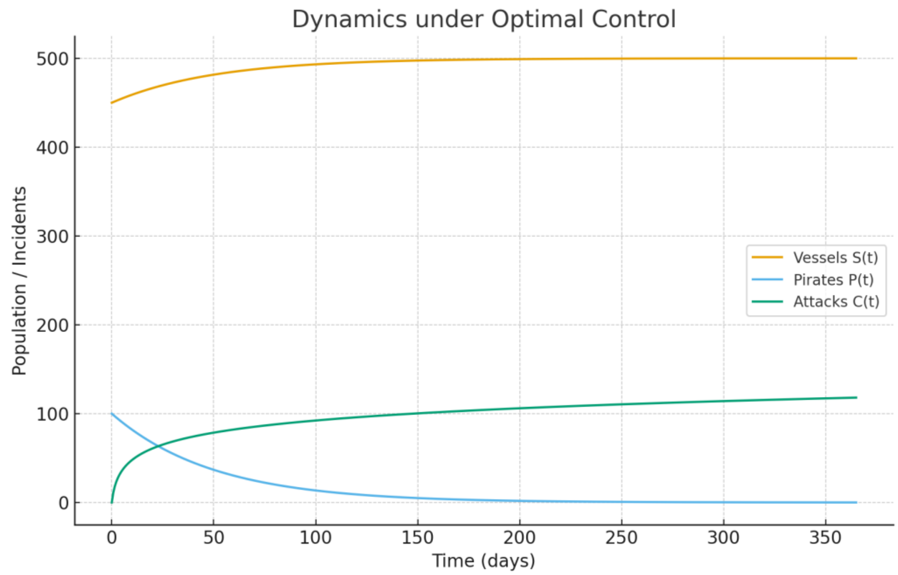

Figure 3: Time evolution under optimal control strategy. Pirate population is suppressed rapidly, vessels remain safe, and cumulative attacks are minimized.

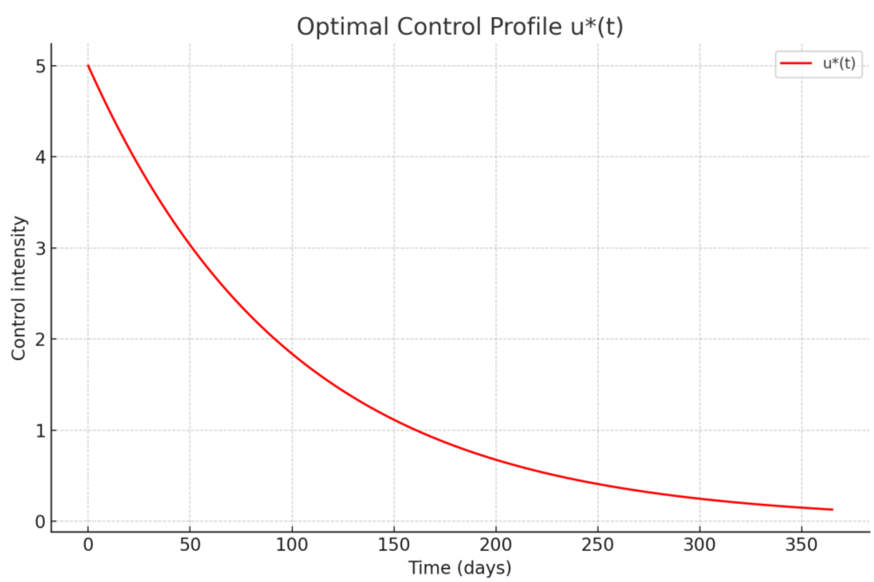

Figure 4: Optimal control profile u*(t) over 365 days. High initial patrol intensity is followed by a gradual reduction as pirate numbers decline.

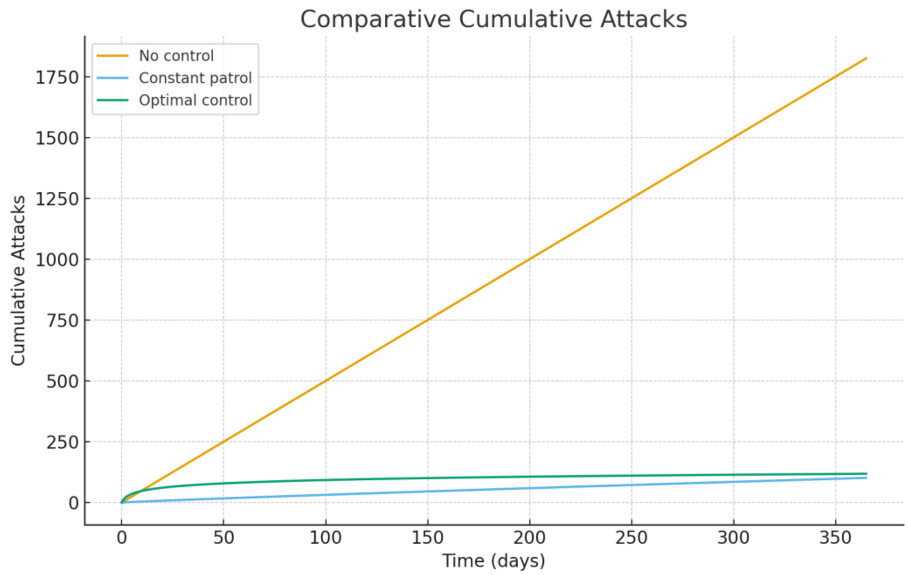

Figure 5: Comparative cumulative attacks under different patrol strategies. Optimal control clearly outperforms both no control and constant patrol.

DISCUSSION OF FINDINGS

1. Effectiveness of Patrols: Patrol resource allocation significantly reduces pirate activity. However, constant patrol is less efficient than optimal adaptive deployment.
2. Economic Trade-Offs: The optimal strategy balances the cost of patrol deployment against the reduction in piracy, showing that security can be achieved without excessive spending.
3. Analytical Insight via HAM: The semi-analytical solutions obtained using the Homotopy Analysis Method (HAM) highlight threshold values of patrol effectiveness k. Above a critical k, piracy collapses rapidly.(Ejinkonye 2021).
4. Policy Implications: Governments and international organizations can reduce marine insecurity by applying adaptive patrol strategies, allocating resources more heavily in critical periods and scaling down later.

SUMMARY

The numerical simulations of the proposed nonlinear optimal control model for marine insecurity are presented. Results demonstrated that:

- In the absence of control, piracy flourishes.
- Constant patrols mitigate piracy but incur high costs.
- Optimal control strategies are most effective, balancing economic cost and maritime security.

These findings provide strong justification for adopting mathematical optimal control models in marine security policymaking.

CONCLUSION AND RECOMMENDATIONS

From the mathematical and numerical analysis, the following conclusions were drawn:

1. Uncontrolled piracy escalates insecurity: In the absence of patrols, pirate activity increases exponentially, leading to significant losses.
2. Constant patrols mitigate piracy but lack efficiency: While they reduce pirate numbers, they incur high costs and do not guarantee long-term suppression.
3. Optimal patrol strategies are most effective: Adaptive patrol intensity provides the best trade-off between security and cost. High patrol levels are necessary initially but can be scaled down as pirate activity diminishes.
4. Analytical thresholds exist: Using HAM, critical values of patrol effectiveness were identified, beyond which piracy collapses. This provides useful theoretical guidelines for maritime security agencies.

Hence, the model confirms that mathematical control strategies can significantly enhance decision-making in combating marine insecurity.

RECOMMENDATIONS

Based on the findings, the following recommendations are made:

1. Adopt adaptive patrol strategies: Security agencies should dynamically allocate patrol resources, concentrating efforts during high-risk periods and scaling down later.
2. International collaboration: Since piracy often crosses national boundaries, regional cooperation is vital for coordinated patrols and resource sharing.
3. Policy integration of mathematical models: Governments should incorporate optimal control models into marine security planning to guide efficient resource allocation.
4. Capacity building: Training naval officers and policymakers in applied mathematical modeling can improve the interpretation and implementation of model-based recommendations.

Future Research: The model can be extended to include stochastic disturbances (e.g., weather, intelligence uncertainty), multi-patrol zones, and socio-economic factors that drive piracy recruitment.

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