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# LAPLACE DECOMPOSITION ANALYSIS OF A RADIATIVE SODIUM BICARBONATE NANO FLUID FLOW NEAR A STAGNATED POINT IN A POROUS MEDIUM

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***Abstract***
Laplace Decomposition Method was used to analyzed the sodium bicarbonate nano fluid near a stagnated point in a porous medium. The Navier Stokes equation describing the flow was transformed into partial differential equations using similarity transformational variables, with boundary conditions. The differential equations were solved using the Laplace Decomposition analysis. Results obtained, show that a decrease in $G_r$ and $Da$ leads to increase in the fluid velocity. However, the fluid velocity decreased as magnetic parameter increases, indicating that the magnetic field tends to retard the motion of the fluid. The flow temperature

**Keywords:** Radiative, Sodium Bicarbonate, Laplace Decomposition, Stagnated point, porous media.

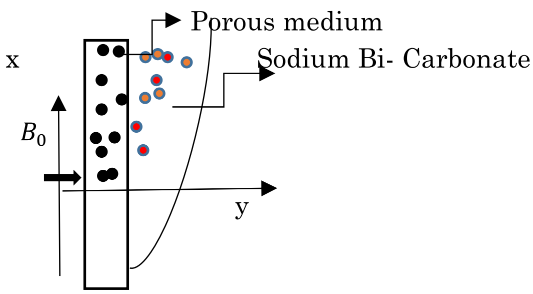
## INTRODUCTION

Improvement in fluid thermal conductivity has gave room to the use of metallic oxide, which occur inform of acidic or basicity in nature. The first appear mostly in electrolysis with discharge of ion, these ions display some level of chemical conductivity. The second one is the metallic oxide, which play vital roles in fluid dynamic. These metal oxides are called nano-particles, forming nano-fluid. Utilizing these particles require increase in the surface area, that is, breaking the surface of such particles into smaller sizes. These particle are introduce into the flowing fluid with the aim of boosting the thermal conductivity of the fluid. Among these metals, is Sodium bi-carbonate $N_{a_2}CO_2$ with the discharge of $N_a^{2+}$ *and* $CO^{2-}$ and balance charge for equal distribution of fluid heat flow. In other word, metallic oxides particles in fluid, sum into nanofluid and play vital role in area of application. Many electrical devices, uses was also found to have increase with an increase in the radiative number $N$, resulting from nano particles used for boosting the thermal conductivity of the fluid.

nanoparticle for heat transfer. Some of which are, transistors, rectifiers, amplifiers and so on. These nanoparticles gain heat and radiate due to the presence of both mixture of ions. Chio (1995) coined the word nanofluid, as a fluid mixture comprising of nano-particles or fibres, which led to the beginning of the studies on nanofluid. The theory of nanofluid has presented many fundamental properties with the enhancements in thermal conductivity as compared to the base fluid, Fan and Wang (2011). Many authors and researchers have carried out theoretical and experimental investigation on the concept of thermal conductivity of nanofluids. Dae-Hwang (2007), Kleinstrtreuer and Feng (2011), Gireesha and Rudraswamy (2014). Hassani (2011), investigated the boundary layer flow problem of a nanofluid past a stretching sheet analytically, using the Homotopy Analysis Method. Both the effect of Brownian motion and thermophoresis were considered in this case. A study on free convective flow over a vertical stretching surface was conducted by Wang (1989). Makinde and Ogulu (2008), examined the effect of thermal radiation on heat and mass transfer of a variable viscosity fluid permeated with a uniform magnetic field. Dash, Mishra and Pattnaik (2021), studied the influence of radiative heat energy on the MHD flow of Cu-Kerosene Nanofluid over a Vertical plate with the use of Laplace transform technique. Grubka and Bobba (1985) investigated fluid flow and heat transfer characteristics on stretching sheet with variable temperature condition. Haque et-al (2011), in their work - MHD free convective heat generating unsteady micro polar fluid flow through a porous medium with constant heat and mass fluxes - observed, that increase of magnetic force number (M) leads to decrease in velocity for an externally cooled plate which indicate that, the magnetic field tends to retard the motion of fluid. Meanwhile, increase in Grashof $(G_r)$ number and strong Darcy$(D_a)$ leads to decrease in velocity. Conclusively, decrease in magnetic number leads to increase in velocity while decrease in Darcy$(D_a)$ leads to an increase in velocity. Laplace decomposition is a numerical scheme which enables one to decomposed the non-linear terms in a given differential equation. Laplace transformation is an approach for solving ordinary differential equation with initial condition, without the consideration of the non-linear term. Decomposed Laplace give advance of the former, because it offered the approach of overcoming non-linear. This advantage enables adoption of the scheme into fluid mechanics. Khuri (2001) and (2004), Yusufoglu (2006) introduce the concept of Laplace Decomposition method which involves Laplace transformation numerical scheme, based on the decomposition method for solving non-linear differential equations. The aim of this work is to apply Laplace Decomposition Method as solution to Radiative Sodium Bicarbonate Nano fluid flow near a stagnated point in a porous medium. However, the effect of $N_{a_2}CO_2$ on fluid flow wasn't considered in details rather but used as a thermal heat booster.

## ASSUMPTIONS

Fluid is considered a steady continuum Newtonian electrically conducting sodium bi-carbonate nano-fluid, with electrical conductivity $\sigma$. It generates or absorbs heat at uniform rate with magnetic Reynolds number ($R_m$ << 1) so that the induced magnetic flied is neglected. It is two-dimensional flow with permeable plate subjected to arbitrary heat flux $q(x)$. The Cartesian coordinate has its origin at the Centre of the plate with $x-axis$ measured along the vertical direction, while the y$-axis$ is in the horizontal direction, a uniform magnetic field is applied in the y$-axis$, gravitational field $g$ is directly opposite to the $x - axis$ with flow velocity $u$ in the vertical direction, as in the diagram below




Due to magnetic effect on flow, Maxwell's equation is given as:

$\nabla.D = \rho_e$

$\nabla \times \mathrm{H} = J + \frac{\partial D}{\partial t}$

$\nabla \times \mathrm{E} = -\frac{\partial B}{\partial t}$

$\nabla.B = 0$

$\frac{\partial \rho_e}{\partial t} + \nabla.J = 0$

Where

H = Magnetic field strength

E = Electric field

J = Current density

With negligible current displacement $\frac{\partial D}{\partial t}$

$\nabla \times \mathrm{H} = J$

The expression of $J$ according to Ohm's law

$J = \frac{E}{\rho} = \sigma E' = \sigma(E + u \times \mathrm{B})$

B = Magnetic induction vector

Applying the non-relativistic Lorentz transformation under magneto hydro dynamic approximation, equation of motion, continuity, energy yield

Continuity equation

$\nabla.v = 0$ 1.00

**Motion equation**

$\rho(u.\nabla)u = -\nabla \mathrm{p} - \rho\nabla\varphi + \mu\nabla^2 + J \times B - \frac{\mu}{k}u$ 1.01

Energy equation

$\rho c_p(\nabla.\mathrm{T})u = -\rho\nabla.\mathrm{u} - \nabla \mathrm{q} + \emptyset + \nabla q_r$ 1.02

The Buoyancy force acting on the fluid is a combination of density gradient and the body force due to gravitational filed

$\rho\nabla\varphi = \rho_\infty \mathrm{g} - \rho \mathrm{g} = \mathrm{g}[\rho_\infty - \rho]$

Density difference is express using volumetary thermal expansion

$\beta = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right) = \frac{-1}{\rho}\left(\frac{\rho_\infty - \rho}{T_\infty - T}\right)$

$\rho_\infty - \rho = -\beta\rho(T_\infty - T)$

$\rho\nabla\varphi = \beta\rho g(T_\infty - T)$

$J \times B = \sigma(E + u \times \mathrm{B}) \times B = B^2\sigma u$

$-\nabla q = -\nabla(-k_T\nabla T) = k_T\nabla^2 T$

Both motion and energy equation yield

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{-1}{\rho}\frac{dp}{dx}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{\beta\rho g(T_\infty-T)}{\rho}-\frac{\sigma B_0^2 u}{\rho} \quad 1.03$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_T}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{Q(T_\infty-T)}{\rho c_p}+\frac{1}{\rho C_\rho}\frac{\partial q_r}{\partial y} \quad 1.04$$

**Boundary conditions**

$u = v = 0,\quad \frac{\partial T}{\partial y}=\frac{q_w x}{k}$

$u = v = ax,\ T = T_w$

1.05

The radiative heat flux term is simplified by using the Roseland diffusion approximation accordingly

$$q_r = -\frac{16\sigma^* T_\infty^3}{3\alpha^*}\frac{\partial T}{\partial y} \quad 1.06$$

Where $\sigma^*$ is the Stefan-Boltzmann constant, $\alpha^*$ is the Rosseland mean absorption coefficient. Substituting (1.06) into (1.04), we have;

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_T}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{Q(T_\infty-T)}{\rho c_p}-\frac{16\sigma^* T_\infty^3}{3\alpha^*\rho C_\rho}\frac{\partial^2 T}{\partial y^2} \quad 1.07$$

To reduce the motion and energy equation to dimensionless form, we introduce similarity variables and dimensionless quantity as ;

$u=\frac{\partial\varphi}{\partial y}, v=-\frac{\partial\varphi}{\partial x},\ \varphi=\sqrt{va}\,xf(\xi,\eta)$ ,

$\eta=\sqrt{\frac{a}{v}}\,y, \xi=\frac{g\beta q}{a^2 k}R_{ex}^{\frac{-1}{2}}, T=\left(\frac{q_w x}{k}\right)R_{ex}^{\frac{-1}{2}}\theta+T_\infty$

$$M^2=\frac{\sigma\beta_0^2}{\alpha\rho},\quad P_r=\frac{v}{a},\ \lambda=\frac{Q}{\alpha\rho C_\rho},\ G_r=\frac{\beta g q}{\alpha\kappa x^2},\ D_a=\frac{1}{k},\ N=\frac{16\sigma^* T_\infty^3}{3\alpha^*\alpha\rho C_\rho}, R_{ex}=\frac{u_\infty x}{v}=\frac{ax^2}{v} \quad 1.08a$$

For dimensionalisation of fluid flow, we used the following;

$$\left.\begin{aligned}
u&=axf'(\eta)\\
\frac{\partial u}{\partial x}&=af'(\eta)\\
\frac{\partial u}{\partial y}&=ax\sqrt{\frac{a}{v}}f''(\eta)\\
\frac{\partial^2 u}{\partial y^2}&=\frac{a^2 x}{v}f'''(\eta)\\
v&=-\sqrt{va}\,f(\eta)\\
\frac{\partial T}{\partial x}&=\frac{q_w}{\kappa}R_{ex}^{\frac{-1}{2}}\theta(\eta)\\
\frac{\partial T}{\partial y}&=\frac{q_w x}{\kappa}\sqrt{\frac{a}{v}}R_{ex}^{\frac{-1}{2}}\theta'(\eta)\\
\frac{\partial^2 T}{\partial x^2}&=0\\
\frac{\partial^2 T}{\partial y^2}&=\frac{q_w x\alpha}{v\kappa}R_{ex}^{\frac{-1}{2}}\theta''(\eta)
\end{aligned}\right\} \quad 1.08b$$

Substituting (1.08a) along with (1.08b) into (1.03) and (1.07) we have,

$$f'''-f'^2+ff''+P_rG_r\theta-[M^2+P_rD_a]f'=0 \quad 1.09$$

$$\theta''-\frac{1}{[1-N]}[P_rf'\theta+P_rf\theta'+P_r\lambda\theta]=0 \quad 1.10$$

With the transformed boundary conditions

$f(0)=0,\ f'(0)=0,\ \theta'(0)=1\ at\ \eta=0$

$f'(\infty)=1\ \ \theta(\infty)=0\ at\ \eta=\infty$ 1.12

**LAPLACE DECOMPOSITION**

Given a non-linear differential equation

$L[v(t)] + [Rv(t)] + [Nv(t)] = g(t)$ 2.01

With initial conditions

$v(0) = h(t),\ v'(t) = s(t), v''(t) = d(t)$ 2.02

Taking the Laplace transformation of equation (2.01), where $L\ operator$ is the order of the differential equation, if ordinary, $[Rv(t)]\ and\ [Nv(t)]$ are linear and non-linear terms of $g(t)$

The general Laplace transform is given as

$s^n l[u(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - s^{n-4}f'''(0) + \cdots + f(0) = 0$ 2.03

Taking the inverse of (2.03)

$u(t) = l^{-1}\left[\frac{1}{s^n}\left(s^{n-1}f(0) + s^{n-2}f'(0) + s^{n-3}f''(0) + s^{n-4}f'''(0) \ldots f^{n-1}(0)\right)\right]$ 2.04

Use this property, take

$l[v(t)] = \frac{h(t)}{s} + \frac{s(t)}{s^2} + \frac{d(t)}{s^3} + \frac{lg(t)}{s^3} - \frac{l[Rv(t)]}{s^3} - \frac{l[Nv(t)]}{s^3}$ 2.05

Replacing the series with infinite series, we have;

$l|\sum_{i=0}^{\infty} v_i(t)| = \frac{h(t)}{s} + \frac{s(t)}{s^2} + \frac{d(t)}{s^3} + \frac{lg(t)}{s^3} - \frac{l[Rv(t)]}{s^3} - \frac{1}{s^3} l|\sum_{i=0}^{\infty} B_i(t)|$ 2.06

Taking inverse Laplace transform of (2.06)

$l^{-1}l|\sum_{i=0}^{\infty} v_i(t)| = l^{-1}\left[\frac{h(t)}{s} + \frac{s(t)}{s^2} + \frac{d(t)}{s^3} + \frac{lg(t)}{s^3} - \frac{l[Rv(t)]}{s^3}\right] - l^{-1}\left[\frac{1}{s^3} l|\sum_{i=0}^{\infty} B_i(t)|\right]$

$\sum_{i=0}^{\infty} v_i(t) = l^{-1}\left[\frac{h(t)}{s} + \frac{s(t)}{s^2} + \frac{d(t)}{s^3} + \frac{lg(t)}{s^3} - \frac{l[Rv(t)]}{s^3}\right] - l^{-1}\left[\frac{1}{s^3} l|\sum_{i=0}^{\infty} B_i(t)|\right]$ 2.07

**DECOMPOSITION OF NON-LINEAR TERM**

The non-linear term is the term with either polynomial more than degree two (2) or have a multiplicative relationship between its derivatives and dependent variable.

If the non-linear term is of the form $u^m\ with\ m\ degree\ polynomial$ , then it's decomposed polynomial is given as:

$B_k = \frac{1}{k!}\ \frac{d^k}{d\lambda^k}\left[N\left(\sum_{i=o}^{k} \lambda^i \psi_i\right)\mathrm{I}_{\lambda=0}\right]$

$B_k = \frac{1}{k!}\ \frac{d^k}{d\lambda^k}[N(\lambda^0\psi_0 + \lambda^1\psi_1 + \lambda^2\psi_2)\mathrm{I}_{\lambda=0}]$

$B_0 = \frac{1}{0!}\ \frac{d^0}{d\lambda^0}[N(\psi_0)] = N(\psi_0)$

$B_1 = \frac{1}{1!}\ \frac{d^1}{d\lambda^1}[N(\lambda^0\psi_0 + \lambda^1\psi_1 + \lambda^2\psi_2)(\psi_1 + 2\lambda\psi_2)\mathrm{I}_{\lambda=0}] = N'(\psi_0)\psi_1$

Proof.

$B_2 = \frac{1}{2!}\ \frac{d^2}{d\lambda^2}\left[N\left(\sum_{i=o}^{2} \lambda^i \psi_i\right)\mathrm{I}_{\lambda=0}\right] = \frac{1}{2!}\ \frac{d^2}{d\lambda^2}[N(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2)\mathrm{I}_{\lambda=0}]$

$= \frac{1}{2}\frac{d}{d\lambda}[N'(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2)(\psi_1 + 2\lambda\psi_2)]$

$= \frac{1}{2}[N''(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2)(\psi_1 + 2\lambda\psi_2)^2 + 2\psi_2 N'(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2)]\mathrm{I}_{\lambda=0}$

$B_2 = N'(\psi_0)\psi_2 + \frac{1}{2}\ N''(\psi_0)\psi_1^2$

$B_3 = \frac{1}{3!}\ \frac{d^3}{d\lambda^3}\left[N\left(\sum_{i=o}^{3} \lambda^i \psi_i\right)\mathrm{I}_{\lambda=0}\right] = \frac{1}{3!}\ \frac{d^3}{d\lambda^3}[N(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3)\mathrm{I}_{\lambda=0}]$

$= \frac{1}{3!}\ \frac{d^2}{d\lambda^2}[N'(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3)(\psi_1 + 2\lambda\psi_2 + 3\lambda^2\psi_3)\mathrm{I}_{\lambda=0}]$

$= \frac{1}{3!}\frac{d}{d\lambda}[N''(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3)(\psi_1 + 2\lambda\psi_2 + 3\lambda^2\psi_3)^2 + N'(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3)(2\psi_2 + 6\lambda\psi_3)\mathrm{I}_{\lambda=0}]$

$= \frac{1}{3!}[N'''(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3)(\psi_1 + 2\lambda\psi_2 + 3\lambda^2\psi_3)^3 + 2N''(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3)(2\psi_2 + 6\lambda\psi_3) + N''(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3)(\psi_1 + 2\lambda\psi_2 + 3\lambda^2\psi_3)(2\psi_2 + 6\lambda\psi_3) + N'(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3)(6\psi_3)\mathrm{I}_{\lambda=0}]$

$B_3 = N'(\psi_0)\psi_3 + N''(\psi_0)\psi_1\,\psi_2 + \frac{1}{6}\,N'''(\psi_0)\psi_1^3$

$B_4 = \frac{1}{4!}\,\frac{d^4}{d\lambda^4}[N(\sum_{i=o}^{4}\lambda^i\psi_i)\mathrm{I}_{\lambda=0}] = \frac{1}{4!}\,\frac{d^4}{d\lambda^4}[N(\lambda^0\psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \lambda^3\psi_3 + \lambda^4\psi_4)\mathrm{I}_{\lambda=0}]$

$B_4 = N'(\psi_0)\psi_4 + N''(\psi_0)\left(\frac{\psi_2^2}{2} + \psi_1\psi_3\right) + \frac{\psi_1^2\psi_2}{2}\,N'''(\psi_0) + \frac{\psi_1^4}{4!}N''''(\psi_0)$

Case 2: when either $[Rv(t)]$ $or$ $[Nv(t)]$ $appear\ in\ the\ form\ ff''\ or\ ff'$
Then $ff''\ or\ ff'$is decompose using Adomian decompose polynomial
$B_k = \sum_{r=0}^{k} f_r f_{k-r}''$
As we proceed, then we can write
$B_0 = f_0 f_0''$
$B_1 = f_0 f_1'' + f_1 f_0''$
$B_2 = f_0 f_2'' + f_1 f_1'' + f_2 f_1''$
$B_3 = f_0 f_3'' + f_1 f_2'' + f_2 f_1'' + f_3 f_0''$
Case 3: when either $Ng\ or\ Rh\ appear\ in\ the\ form\ {f''}^2\ or\ {f'}^2,$
The above nonlinear term as appear in case three, can be written as
$(f'')^2 = f''f''$
Then ${f''}^2\ or\ {f'}^2$is decompose using Adomian decompose polynomial
$B_i = \sum_{k=0}^{i} f_k''\,f_{i-k}''\ \forall i = 0 \dots n$ $B_0 = f_0''f_0''$
$B_1 = f_0''f_1'' + f_1''f_0'' = 2f_0''f_1''$
$B_2 = f_0''f_2'' + f_1''f_1'' + f_2''f_0'' = (f_1'')^2 + 2f_0''f_2''$
$B_3 = f_0''f_3'' + f_1''f_2'' + f_2''f_1'' + f_3''f_0'' = 2f_0''f_3'' + 2f_1''f_2''$
Likewise
$(f')^2 = f'f'$
$B_i = \sum_{k=0}^{i} f_k'\,f_{i-k}'\ \forall i = 0 \dots n$
$B_0 = f_0'f_0' = (f_0')^2$
$B_1 = f_0'f_1' + f_1'f_0' = 2f_0'f_1'$
$B_2 = f_0'f_2' + f_1'f_1' + f_2'f_0' = (f_1')^2 + 2f_0'f_2'$
$B_3 = f_0'f_3' + f_1'f_2' + f_2'f_1' + f_3'f_0' = 2f_0'f_3' + 2f_1'f_2'$

2.2 APPLICATION TO THE FLOW PROBLEM

Consider moment equation in (1.09)
$f''' - {f'}^2 + ff'' + P_rG_r\theta - [M^2 + P_rD_a]f' = 0$
Take the Laplace transform of the moment equation, yield
$s^3L[f] - s^2f(0) - sf'(0) - f''(0) = L\left[{f'}^2\right] - L[ff''] - P_rG_rL[\theta] - [M^2 + P_rD_a]L[f']$
${f'}^2 = \sum_n^\infty A_n \quad ff'' = \sum_n^\infty B_n$
$L[f] = \frac{1}{s}f(0) + \frac{1}{s^2}f'(0) + \frac{1}{s^3}f''(0) + \left(\frac{1}{s^3}\left[L[A_n] - L[B_n] - P_rG_rL[\theta] - [M^2 + P_rD_a]L[f']\right]\right)$ (2.08)
$B_0 = \frac{\beta^2}{2}\eta^2, \quad A_0\ \beta^2\eta^2$

Taking the inverse Laplace transform of 2.08a

$f = \frac{1}{s^3} f''(0) + L^{-1}\left(\frac{1}{s^3}\left[L[A_n] - L[B_n] - P_r G_r L[\theta] - [M^2 + P_r D_a] L[f']\right]\right)$ 2.08b

Let $f''(0) = \beta$ such that its lies within interval of study

$f_0 = \frac{\beta\eta^2}{2},$

$f_1 = \frac{\beta^2}{120}\eta^5 - \frac{P_r G_r}{24}\eta^4 - \frac{[M^2 + P_r D_a]\beta}{24}\eta^4$

Series approximation solution for momentum flow

$f(\eta) = \sum_n^\infty f_n = f_0 + f_1 = \frac{\beta\eta^2}{2} + \frac{\beta^2}{120}\eta^5 - \frac{P_r G_r}{24}\eta^4 - \frac{[M^2 + P_r D_a]\beta}{24}\eta^4$

Taken Laplace transform for energy equation

$\theta'' = \frac{1}{[1-N]}[P_r f'\theta + P_r f\theta' + P_r \lambda\theta]$

2.09

$L[\theta] = \frac{1}{s}\theta(0) + \frac{1}{s^2}\theta'(0) + \frac{1}{[1-N]}\frac{1}{s^2} L[P_r f'\theta + P_r f\theta' + P_r \lambda\theta]$ 2.10

Let $\theta(0) = \alpha$ such that its lies within boundary of study

Take inverse Laplace transform of (2.10)

$\theta(\eta) = \alpha\eta + \eta + \frac{1}{[1-N]} L^{-1}\left(\frac{1}{s^2} L[P_r f'\theta + P_r f\theta' + P_r \lambda\theta]\right)$

$\theta_0 = (\alpha + 1)\eta$

$\theta_{n+1} = \frac{1}{[1-N]} L^{-1}\left(\frac{1}{s^2} L[P_r C_n + P_r D_n + P_r \lambda\theta_0]\right)$

$f'\theta = C_n = \sum_{k=0}^n f_k'\theta_{n-k} \quad f\theta' = D_n = \sum_{k=0}^\infty f_k\theta_{n-k}$

$\theta_1 = \frac{1}{[1-N]} L^{-1}\left(\frac{1}{s^2} L[P_r C_0 + P_r D_0 + P_r \lambda\theta_0]\right)$

$C_0 = \beta[\alpha + 1]\eta^2$

$D_0 = \beta[\alpha + 1]\frac{\eta^2}{2}$

Substituting$C_0$ *and* $D_0$ into $\theta_1$, yield

$\theta_1 = \frac{P_r[\alpha+1]}{[1-N]} L^{-1}\left(\frac{1}{s^2} L\left[\beta\eta^2 + \beta\frac{\eta^2}{2} + \lambda\eta\right]\right)$

$\theta_1 = \frac{P_r[\alpha+1]}{[1-N]}\left[\frac{\beta}{8}\eta^4 + \frac{\lambda\eta^3}{6}\right]$

Choosing $\alpha = 0.02, \beta = 0.31, \lambda = 0.10$

Series approximation solution for energy flow

$\theta(\eta) = \sum_n^\infty \theta_n = \theta_0 + \theta_1 = (\alpha + 1)\eta + \frac{P_r[\alpha+1]}{[1-N]}\left[\frac{\beta}{8}\eta^4 + \frac{\lambda\eta^3}{6}\right]$ 2.11

## RESULTS AND DISCUSSION

Laplace Decomposition method was use to analyzed a radiative sodium bi carbonate fluid, the fluid was investigated at the fixed values of $p_r = 0.71$, $\alpha = 0.02$, $\beta = 0.31$ and $\lambda = 0.10$, at different values of $G_r = 5, 6, 7$. $D_a = M = 1,2,3$, $N = 5,6,7$. Velocity Profile of the fluid is represented by figure 1. It indicated that a decrease in the Grashof number $G_r$ led to an increase in the fluid velocity. In figure 2, a decrease in the Darcy number $D_a$ was found to results in increase in the fluid flow, in agreement with Haque. Figure 3, indicated a decrease in the flow velocity as result of increase in the Magnetic number, which also agreed with Haque. In Figure 4, there is an increase in the radiative parameter N in the presence of Sodium Bicarbonate, which increased the temperature of the fluid flow, in agreement with Gireesha and Rudraswany(2014)

$f(\eta)$

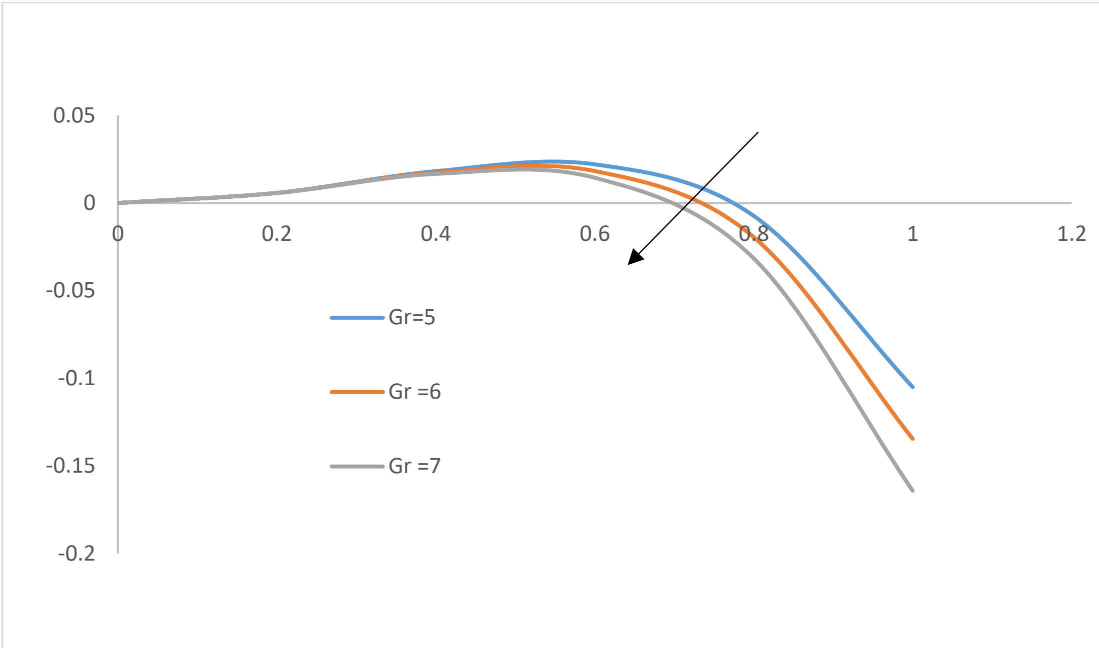
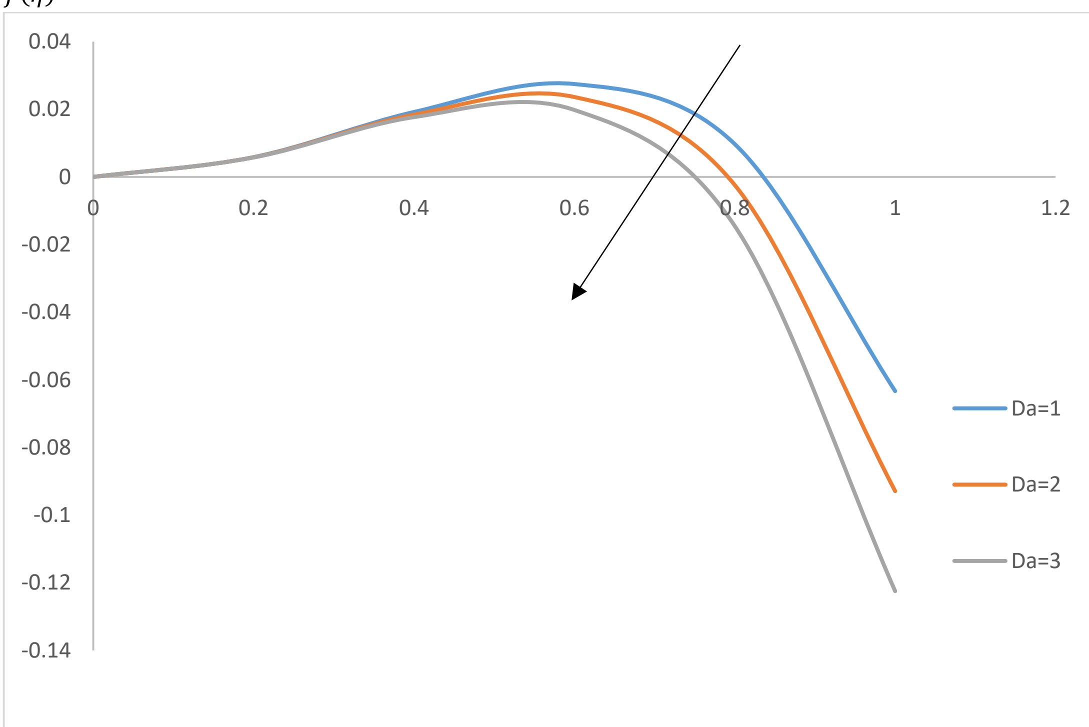



Fig 1. Velocity profile at $P_r = 0.71, M = D_a = 1$

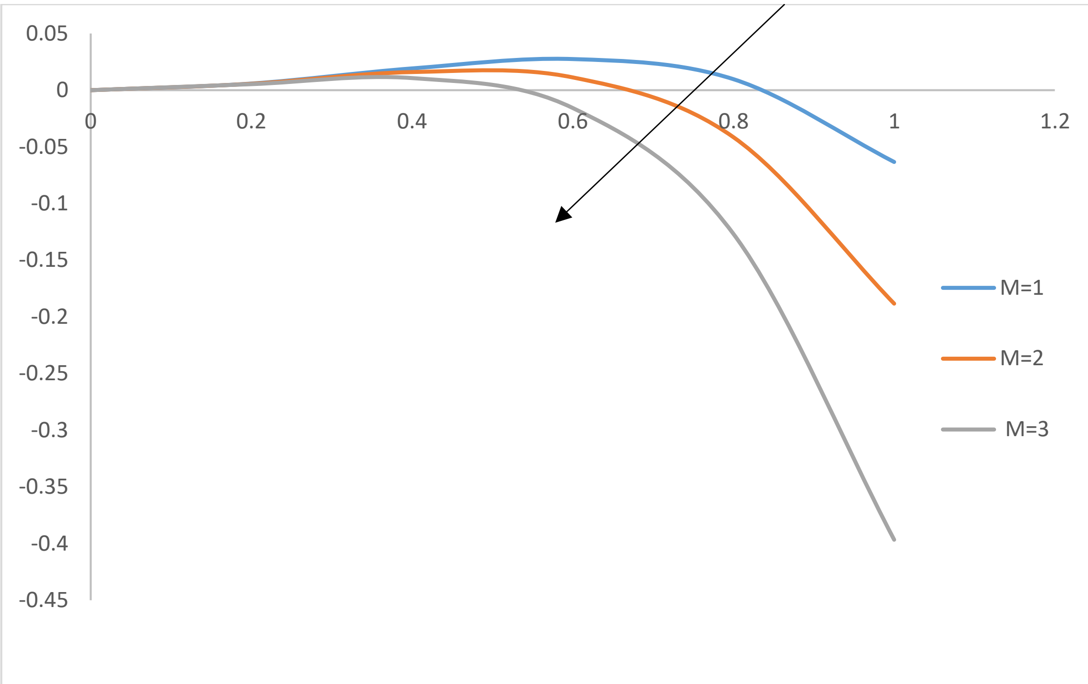
$f(\eta)$




$\eta$

Fig 2 velocity profile at $P_r = 0.71, M = 1, G_r = 5$

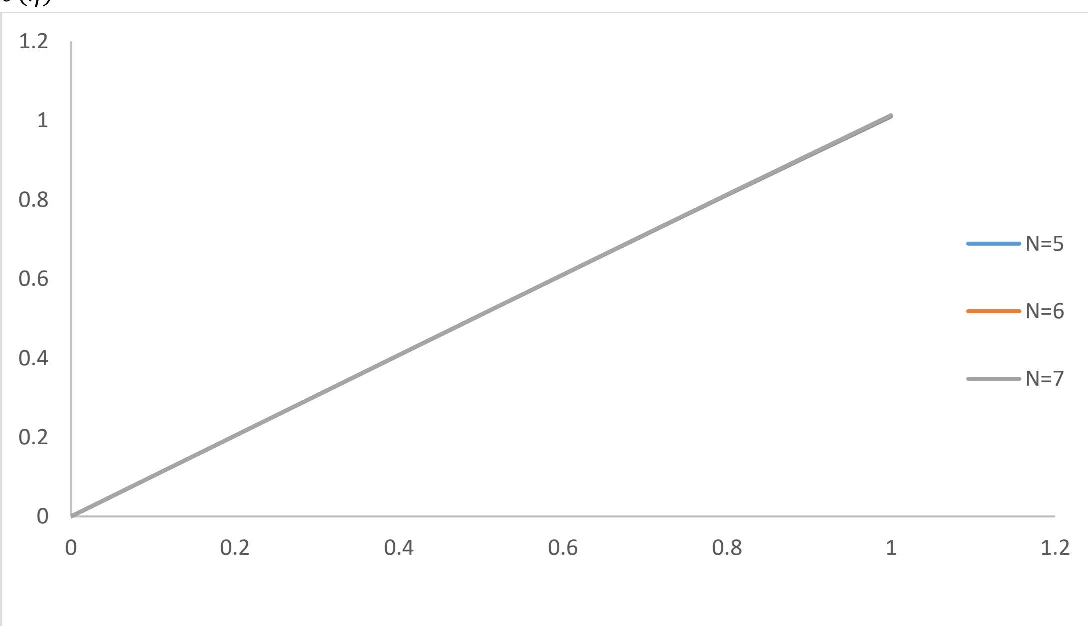
$f(\eta)$




$\eta$

Fig 3 velocity profile at $P_r = 0.71, D_a = 1, G_r = 5$

$\theta(\eta)$




$\eta$

Fig 4 Temperature profile at $P_r = 0.71$

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